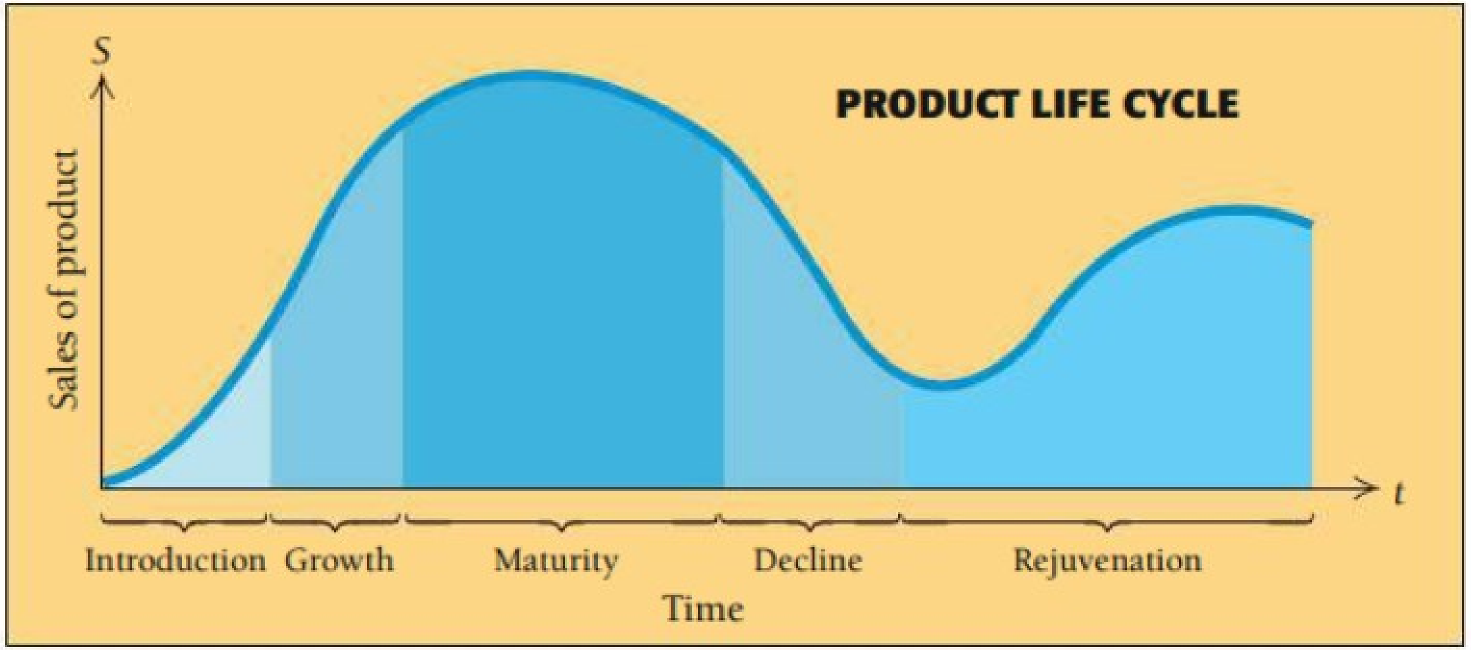
FAR BEYOND

MAT122

Extrema

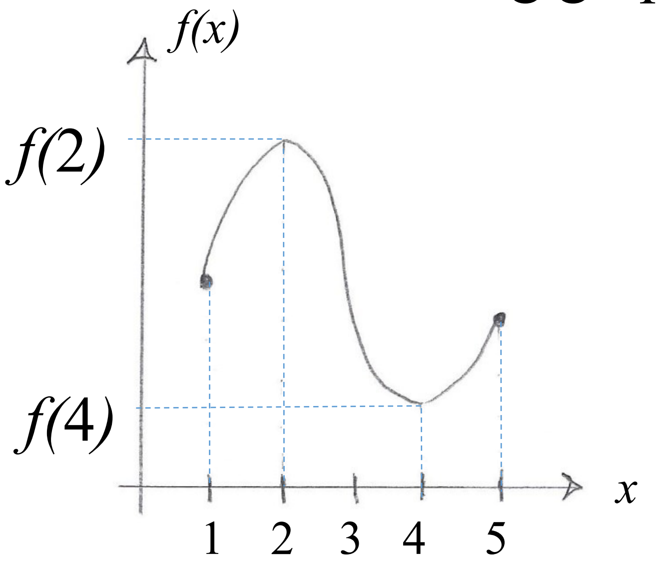
Stony Brook University

Product Life Cycle



Absolute Extrema

consider the following graph:



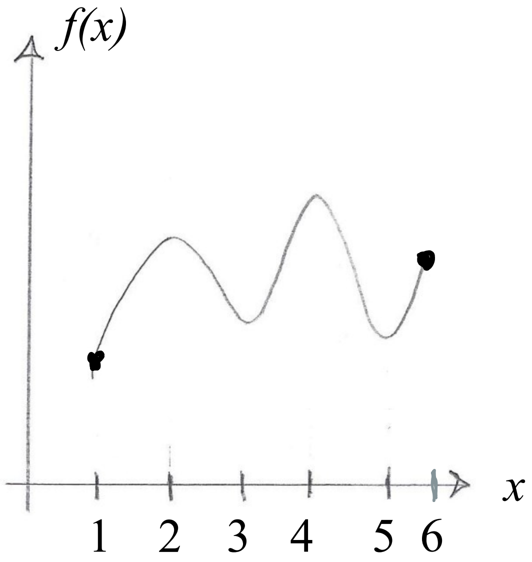
<u>consider $\boldsymbol{x = 2}$ and $\boldsymbol{x = 4}$</u>:

collectively, max/min called <u>**extreme values**</u> or <u>**extrema**</u>

potential max/min called <u>**critical points**</u>

Local Extrema

sometimes there are multiple minima/maxima in a function



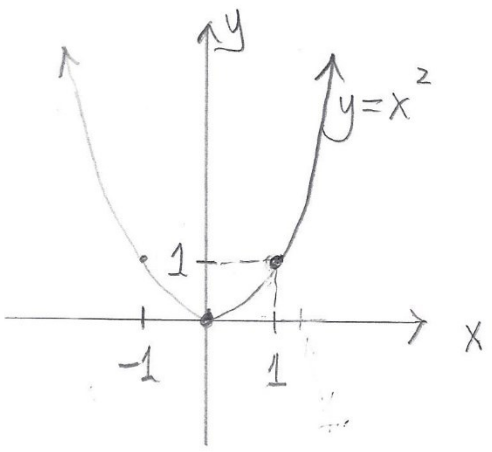
absolute minimum:

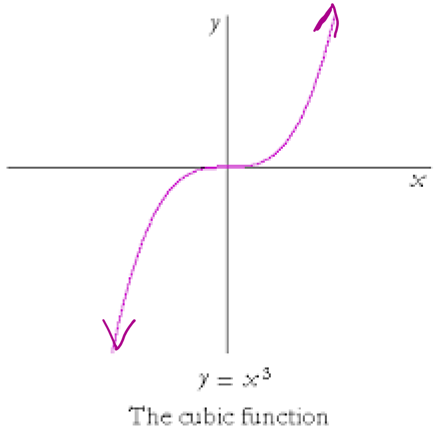
absolute maximum:

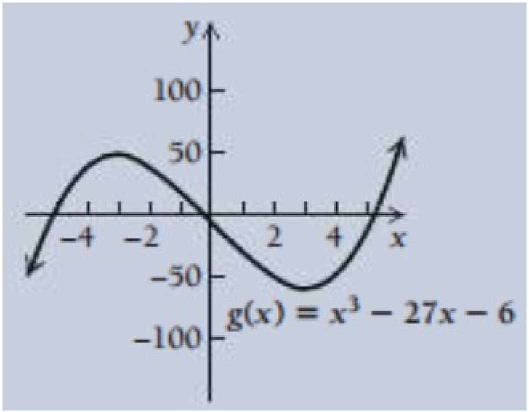
<u>local</u> maximum:

<u>local</u> minima:

Extrema - examples







$$y = x^2$$

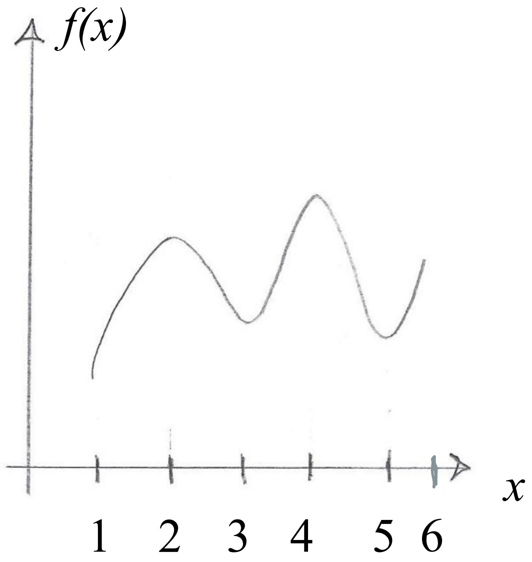
$$y = x^3$$

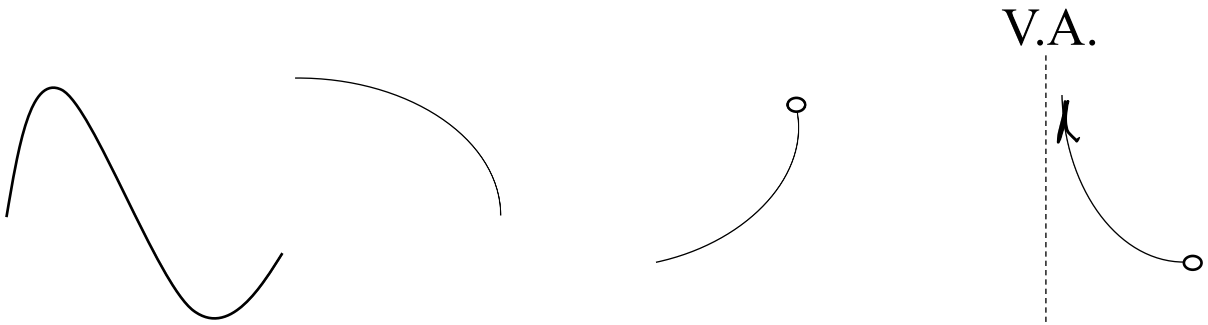
$$g(x) = x^3 - 27x - 6$$

Extrema on a Closed Interval

re-visit previous closed interval graph:

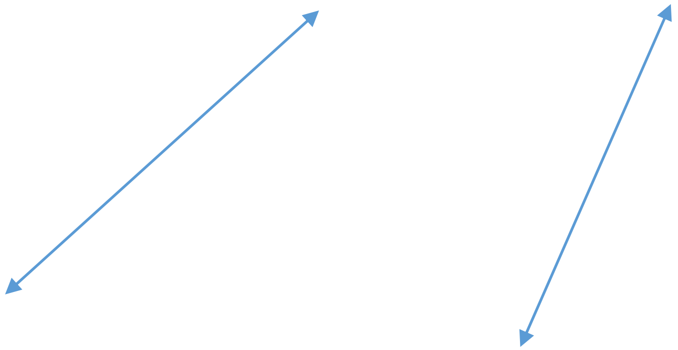
Rule:
If f is continuous on a closed interval, check the endpoints for extrema.





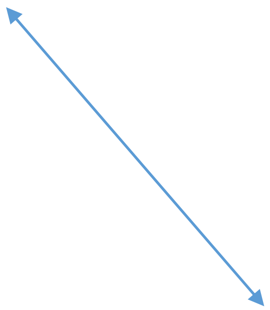
Rate of Change - Refresh

Slope measures the **steepness** of a line or the **rate of change** at a place on a curve



steeper slope implies higher rate of change

(x-values and y-values are both increasing)



negative slope implies negative rate of change

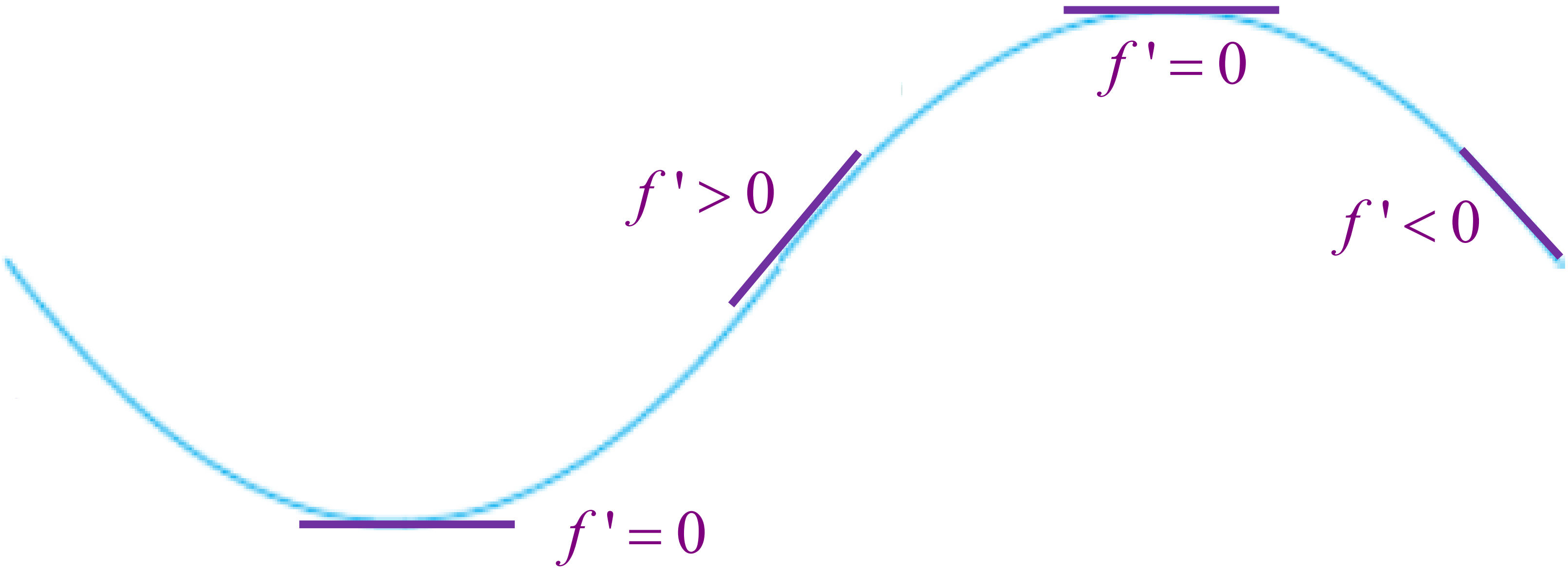
(as x-values are increasing, y-values are decreasing)

slope is 0

NO change

Meanings of Derivatives – Review #1

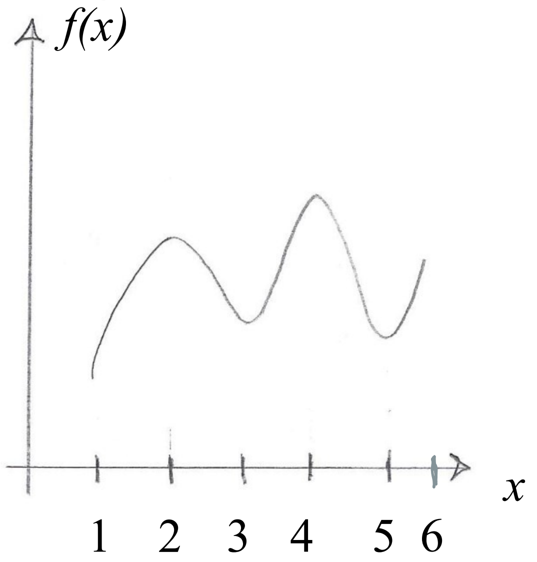
Increasing/Decreasing

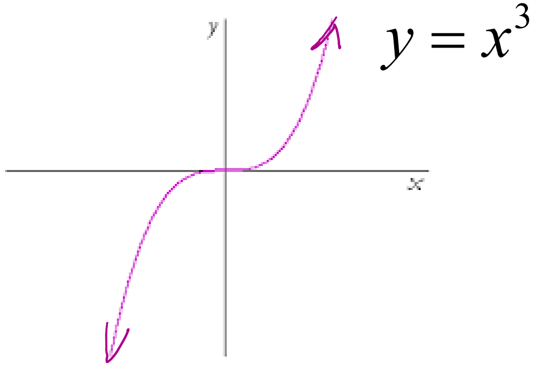


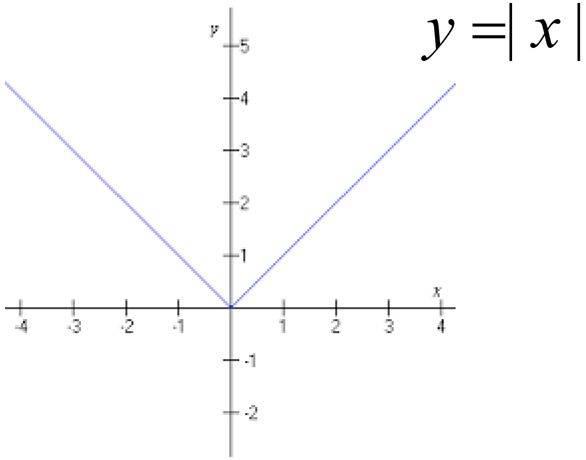
Local Extrema and Slope of Tangent Line

anywhere there is local extrema,
the slope of its tangent line is 0:

<u>Fermat's Theorem</u>:
If *f* has a local max or min at *c* and *f'(c)* exists, then *f'(c)* = 0.





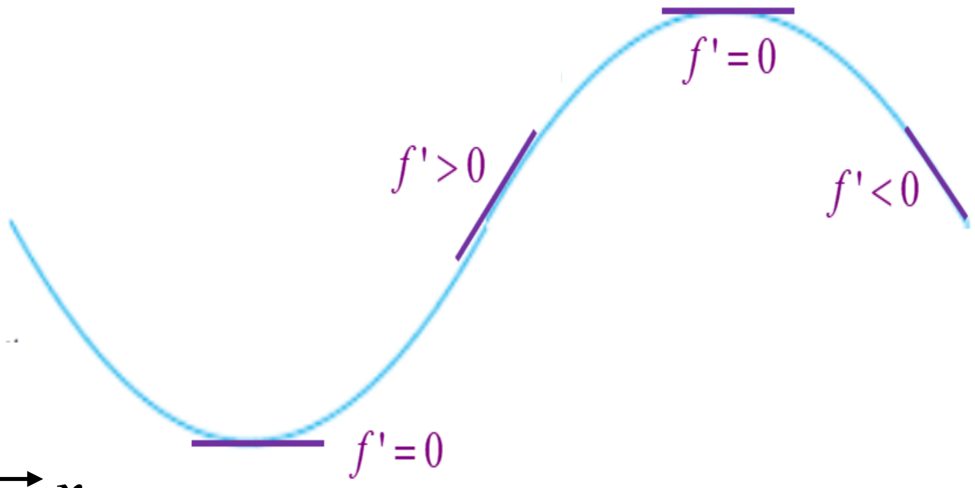


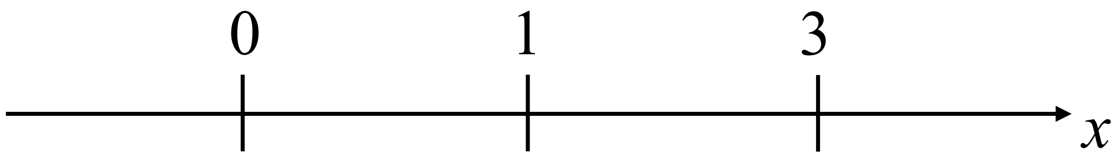
Find Extrema w Differentiation – all reals

ex. Identify extrema of $f(x) = 3x^4 - 16x^3 + 18x^2$

Get critical points by setting first derivative equal to 0.

Do: $f'(x) =$





Find Extrema – revisit with Closed Interval

ex. Identify extrema of $f(x) = 3x^4 - 16x^3 + 18x^2$ on $-1 \le x \le 4.$

Get critical points by setting first derivative equal to 0.

$$f'(x) = 12x^3 - 48x^2 + 36x = 0$$
$$12x\left(x^2 - 4x + 3\right) = 0$$
$$12x(x-1)(x-3) = 0$$

To find **absolute** extrema:

plug **critical points** and **ENDPOINTS** into original function: